THERMAL OSCILLATION CONVECTION IN A LAYER OF LIQUID WITH WEIGHTLESSNESS OR REDUCED GRAVITATION

S. M. Zen'kovskaya and S. N. Ovchinnikova UDC 536.25

Oscillating convection is studied in a layer of viscous incompressible liquid which is in a high-frequency vibration field. Analysis is carried out on the basis of averaged Boussinesq equations obtained in [1, 2]. Cases are considered of total weightlessness and low gravitation.

Secondary regimes are studied numerically which arise in the vicinity of the critical Reyleigh number. Types of loss of stability are studied, i.e., soft or hard in relation to direction and the vibration velocity. Smooth and rapid velocity and temperature components are calculated which are used in working out average characteristics of heat transfer and kinetic energy. It appears that with different vibration directions loss of stability may lead to both an increase and to a reduction in energy. It is found that with all of the vibration directions the average Nusselt number increases with an increase in supercriticality. In the case of reduced gravitation values of the oscillation parameter are found with which there is emergence into weightlessness.

1. The problem of the effect of vertical high-frequency oscillations on the occurrence of convection in a nonisothermal liquid is studied in [1] where an averaging method of the Kapitsa form is used in convection equations in a Boussinesq approximation. By separating movement into smooth and oscillating parts a closed set of equations is derived in [1] for averaged velocity and temperature fields; the "rapid" component is expressed explicitly in terms of the "slow" component. Convection is characterized by three dimensionless parameters: Prandtl $\Pr = \nu/\chi$ and Rayleigh $R = (T_1 - T_2)\beta g \ell^3/(\chi\nu)$ numbers, and a vibration analog of the Rayleigh number $\mu = (T_1 - T_2)^2\beta^2 a^2 \ell^2/(2\nu\chi)$. In [1, 3] it is established that by means of vertical oscillations it is possible to provide convection immediately for all R. It follows from the results of [3] that convection in weightlessness does not arise with vertical oscillations. Starting from [1], the averaging method is used in a whole series of works in studying convection in a high-frequency vibration field. It is noted that the method was first applied to equations in partial derivatives by V. N. Chelomei in studying the dynamic stability of elastic systems under the action of vibration. A rigorous mathematical substantiation of the averaging method is given in [4, 5] for some classes of infinite dissipative systems, and in particular for the problem of convection with vertical oscillations. This makes it possible to study the asymptotic stability of periodic solutions of the original problem on the basis of analyzing the averaged solutions.

The case is considered in [2, 6] of vibration of an arbitrary direction: averaged equations are derived and gravitational convection is studied. It is shown tht for any vibration direction, excluding the vertical, convection may arise both with heating from below and with heating from above. In [7, 8] averaged equations from [2] were analyzed in the interest of a special case of convection in weightlessness, conditions are indicated for existence of an equilibrium condition, and some examples are given. Relationships are given between critical values of parameters assuming that instability is monotonic. The case of a binary mixture is studied in [9, 10]. It is found that in a layer of two-component liquid convection in weightlessness also arises with transverse oscillations. Comparison of numerical results [11-13] with asymptotic results from [9, 10, 14] makes it possible to estimate the value of frequency starting from which the averaging method is effective.

A review of work on oscillating convection in weightlessness may be found in [15]. Only those works are mentioned above which are connected with the method of averaging. The main results for application and substantiation of this method in the problem of thermal oscillating convection are given in [16, 17]. Experimental proof is obtained in [18] for the effects of high-frequency vibration in studying convective instability.

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2. We consider a plane horizontal layer ($-l/2 \leqslant z \leqslant l/2, -\infty < x < +\infty$) of viscous incompressible liquid at whose boundaries temperature is prescribed. The layer as a single unit performs harmonic oscillation along an axis with a unit vector $\mathbf{s} = (\cos\varphi, \sin\varphi)$, where angle φ is read from the direction along the layer. It is assumed that oscillation frequency ω is high ($\omega \to \infty$), and the amplitude a/ω is low. In a coordinate system connected with the oscillating layer convection equations in the Boussinesq approximation have the form

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}, \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu\Delta\mathbf{v} - \beta(\mathbf{g} + a\omega\cos\omega t\mathbf{s})T,$$
$$\partial T/\partial t + (\mathbf{v}, \nabla T) = \chi\Delta T, \ \operatorname{div}\mathbf{v} = 0, \tag{2.1}$$
$$\mathbf{v} = 0 \text{ with } z = \pm l/2, \ T = T_1 \text{ with } z = -l/2, \ T = T_2 \text{ with } z = l/2.$$

here $\mathbf{g} = (0, -g)$; g is free fall acceleration; $\mathbf{v}$ is relative velocity of liquid movement; T, ρ are temperature and density; ν, χ, β are coefficients of kinematic viscosity, thermal diffusivity, and thermal expansion.

Problem (2.1) has a $2\pi/\omega$-periodic solution with respect to time

$$\mathbf{v}_0 = (v_{0x}(z, t), 0), \ T_0 = -Cz + B, \tag{2.2}$$
$$p_0 = \rho\beta(g - a\omega\cos\omega t\sin\varphi)\left(-\frac{Cz^2}{2} + Bz\right) + \text{const},$$

where

$$C = (T_1 - T_2)/l, \ B = (T_1 + T_2)/2,$$
$$v_{0x}(z, t) = F_1(z)\sin\omega t + F_2(z)\cos\omega t,$$
$$F_1(z) = A_1 \operatorname{ch}\omega_1 z \sin\omega_1 z + B_1 \operatorname{sh}\omega_1 z\cos\omega_1 z + Ca\beta\cos\varphi z,$$
$$F_2(z) = A_1 \operatorname{sh}\omega_1 z\cos\omega_1 z - B_1\operatorname{ch}\omega_1 z\sin\omega_1 z,$$
$$B_1 = \frac{Ca\beta\cos\varphi\operatorname{sh}(\omega_1 l/2)\cos(\omega_1 l/2)}{2\left(\operatorname{ch}^2(\omega_1 l/2)\sin^2(\omega_1 l/2) + \operatorname{sh}^2(\omega_1 l/2)\cos^2(\omega_1 l/2)\right)},$$
$$A_1 = B_1\frac{\operatorname{ch}(\omega_1 l/2)\sin(\omega_1 l/2)}{\operatorname{sh}(\omega_1 l/2)\cos(\omega_1 l/2)}, \quad \omega_1 = \sqrt{\omega/(2\nu)}.$$

The solution obtained has an uneven velocity profile with respect to z typical for layers of mixing. With absence of vibration (a = 0) or with vertical vibrations ($\varphi = \pi/2$) this solution corresponds to the assumption of mechanical equilibrium. It follows from (2.2) that with $\varphi \neq \pi/2$ and with existence of a temperature difference at the boundaries mechanical equilibrium is impossible.

The averaging method makes it possible to study the stability of periodic solution (2.2) with a study of the stability of the average $2\pi/\omega$ stationary solution with respect to a time interval satisfying the set of averaged equations.

In order to derive this set we investigate the solution of $\mathbf{v}$, T, p in the form

$$\mathbf{v} = \mathbf{u} + \boldsymbol{\xi}, \ T = \tau + \eta, \ p = q + \delta, \tag{2.3}$$

where $\mathbf{u}$, τ, q are slow components, and ξ, η, δ are rapid components depending on rapid time ωt. In the time interval $2\pi/\omega$ rapid components have a zero average, and slow components are assumed to be stationary. Functions ξ, η are expressed [1] in terms of slow components:

$$\boldsymbol{\xi} = -a\beta\sin\omega t\mathbf{w}, \ \eta = -\frac{a\beta}{\omega}\cos\omega t(\mathbf{w}, \nabla\tau) \tag{2.4}$$

($\mathbf{w} = \Pi(\mathbf{s}\tau)$ is an orthoprojector in L_2 for a subspace of solenoidal vectors with a normal component w_n equal to zero at the boundary). By substituting (2.3) in (2.1) and averaging with respect to explicitly contained time we obtain [2, 6] a closed dynamic set for an averaged hydrodynamic field:

$$\frac{\partial\mathbf{u}}{\partial t} + (\mathbf{u}, \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla q + \nu\Delta\mathbf{u} - \beta\mathbf{g}\tau + \frac{a^2\beta^2}{2}(\mathbf{s}(\mathbf{w}, \nabla\tau) - (\mathbf{w}, \nabla)\mathbf{w}),$$
$$\Delta\mathbf{w} = -\operatorname{rot}\operatorname{rot}(\tau\mathbf{s}), \ \operatorname{div}\mathbf{u} = 0, \ \partial\tau/\partial t + (\mathbf{u}, \nabla\tau) = \chi\Delta\tau, \tag{2.5}$$
$$\tau = T_1 \text{ with } z = -l/2, \ \tau = T_2 \text{ with } z = l/2, \ \mathbf{u} = 0, \ w_n = 0$$
$$\text{with } z = \pm l/2.$$

Problem (2.5) has an accurate stationary solution:

$$\mathbf{u}_0 = 0,\ \tau_0 = -Cz + B,\ \mathbf{w}_0 = (-Cz\cos\varphi,\ 0),\ q_0 = \rho\beta g(-Cz^2/2 + Bz) + \text{const}. \tag{2.6}$$

It is easy ot see that the solution of $\mathbf{u}_0$, τ_0, q_0 is an average solution with respect to period $2\pi/\omega$ from (2.2) and solution (2.3) ($\mathbf{v}_0$, T_0, p_0) corresponding to it gives the main part of the periodic solution (2.2) without taking account of functions of the boundary layer type which appear in (2.2) with $\omega \to \infty$. We find the secondary solution of problem (2.5) distinct from (2.6) in the form

$$\mathbf{u} = \mathbf{u}_0 + \mathbf{v}_1,\ \tau = \tau_0 + T,\ \mathbf{w} = \mathbf{w}_0 + \mathbf{w}_1. \tag{2.7}$$

We substitute (2.7) in (2.5), we move to dimensionless variables, and we introduce a flow function $\psi(x, z, t)$ and $\Phi(x, z, t)$ so that $\mathbf{v}_1 = (\partial\psi/\partial z, -\partial\psi/\partial x)$, $\mathbf{w}_1 = (\partial\Phi/\partial z, -\partial\Phi/\partial x)$. For the unknowns ψ, Φ, T we obtain a nonlinear problem

$$-\frac{\partial\Delta\psi}{\partial t} + \Delta^2\psi - R\frac{\partial T}{\partial x} - \mu\left(\frac{\partial T}{\partial x}\cos^2\varphi + \frac{\partial^2\Phi}{\partial x^2}\sin\varphi - \frac{\partial^2\Phi}{\partial x\,\partial z}\cos\varphi\right) =$$

$$= K(\psi, \Delta\psi) + \mu\left(\frac{\partial}{\partial x}K(\Phi, T)\sin\varphi - \frac{\partial}{\partial z}K(\Phi, T)\cos\varphi + K(\Phi, \Delta\Phi)\right),$$

$$-\Pr\frac{\partial T}{\partial t} + \Delta T - \frac{\partial\psi}{\partial x} = K(\psi, T),\quad \Delta\Phi - \frac{\partial T}{\partial z}\cos\varphi + \frac{\partial T}{\partial x}\sin\varphi = 0, \tag{2.8}$$

$$\psi = \partial\psi/\partial z = T = \Phi = 0 \text{ with } z = \pm 1/2,$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial z^2$, and operator K acts on arbitrary smooth functions u and v according to the rule

$$K(u, v) = \frac{\partial u}{\partial z}\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial z}.$$

Problem (2.8) is characterized by four dimensionless parameters: $\Pr = \nu/\chi$; $R = \beta g C l^4/(\chi\nu)$; $\mu = a^2\beta^2C^2l^4/(2\chi\nu)$ (φ is angle determining the vibration direction).

3. To problem (2.8) we apply the Lyapunov–Shmidt method developed for the problem of hydrodynamics, and in particular for the Yudovich convection problem [19, 20]. We consider the stationary linearized problem corresponding to (2.8):

$$\Delta^2\psi - R\frac{\partial T}{\partial x} - \mu\left(\frac{\partial T}{\partial x}\cos^2\varphi - \frac{\partial^2\Phi}{\partial x^2}\sin\varphi - \frac{\partial^2\Phi}{\partial x\,\partial z}\cos\varphi\right) = 0,$$

$$\Delta T - \frac{\partial\psi}{\partial x} = 0,\quad \Delta\Phi - \frac{\partial T}{\partial z}\cos\varphi + \frac{\partial T}{\partial x}\sin\varphi = 0, \tag{3.1}$$

$$\psi = \frac{\partial\psi}{\partial z} = T = \Phi = 0 \text{ with } z = \pm 1/2.$$

When there is no vibration ($\mu = 0$) it is shown in [20] that in the vicinity of fundamental number R_* of problem (3.1) there is loss of equilibrium stability and with an accuracy up to shear along the layer a single stationary $2\pi/\alpha$-periodic secondary solution of problem (2.8) with respect to x arises which depends analytically on parameter $\varepsilon = \sqrt{R - R_*}$. These results are based on the simplicity of the fundamental number and invariance of the problem in relation to some group of transformations (for numerical solutions see [21]).

Nonlinear problem (2.8) is invariant with respect to groups of shears along the layer $I_h(U(x, z)) = U(x + h, z)$ and transformation of the inversion $L(U(x, z)) = (\psi(-x, -z), -T(-x, -z), \Phi(-x, -z))$, where $U(x, z) = (\psi(x, z), T(x, z), \Phi(x, z))$. Let μ_* be the fundamental number of problem (3.1) with fixed Pr, R, φ, α. The simplicity of μ_* is shown in [10] for $\varphi = 0$, $R = 0$, and in the rest of the cases it was verified numerically. The stationary $2\pi/\alpha$-periodic solution of problem (2.8) with respect to x with $\mu = \mu_* + \delta_1\varepsilon^2$ is found in the form of a Lyapunov–Shmidt series

$$\psi = \sum_{n=1}^{\infty}\psi_n\varepsilon^n,\quad T = \sum_{n=1}^{\infty}T_n\varepsilon^n,\quad \Phi = \sum_{n=1}^{\infty}\Phi_n\varepsilon^n. \tag{3.2}$$

If a solution of (3.2) exists with $\mu > \mu_*$, then $\delta_1 = 1$ (soft loss of stability), and if with $\mu < \mu_*$, then $\delta_1 = -1$ (hard loss of stability). By using the calculation procedure given in detail in [22] we obtain

$$u = \left(\varepsilon\beta_1\frac{\partial\psi_0}{\partial z} + \varepsilon^2\beta_1^2\frac{\partial w_{21}}{\partial z},\ -\varepsilon\beta_1\frac{\partial\psi_0}{\partial x} - \varepsilon^2\beta_1^2\frac{\partial w_{21}}{\partial x}\right) + O(\varepsilon^3),$$

TABLE 1

φ	r	μ_*	α_*	$\delta_1\beta_1^2$	C_1	C_2
0	5	2117,3	3,2262	0,5736	0,5110	—30,376
	10	2123,0	3,2265	0,5698	0,5091	—30,403
	15	2124,9	3,2266	0,5685	0,5084	—30,411
	∞	2128,7	3,2260	0,5661	0,5071	—30,423
10	5	2223,7	3,2081	—0,3301	0,3072	—18,328
	10	2229,9	3,2085	—0,3306	0,3086	—18,500
	15	2231,9	3,2091	—0,3288	0,3073	—18,453
	∞	2236,0	3,2083	—0,3329	0,3116	—18,767
20	5	2597,5	3,1458	—0,0348	0,0371	—2,237
	10	2605,2	3,1460	—0,0343	0,0368	—2,228
	15	2607,8	3,1460	—0,0342	0,0367	—2,227
	∞	2613,0	3,1462	—0,0339	0,0364	—2,219
30	5	3476,0	3,0013	—0,00667	0,00909	—0,5552
	10	3487,8	3,0011	—0,00663	0,00907	—0,5569
	15	3491,8	3,0041	—0,00649	0,00890	—0,5479
	∞	3499,7	3,0008	—0,00659	0,00905	—0,5588
40	5	5706,9	2,6671	—0,00186	0,00367	—0,2165
	10	5730,2	2,6579	—0,00204	0,00401	—0,2379
	∞	5753,7	2,6562	—0,00205	0,00405	—0,2424
50	5	12758,5	1,9836	—0,00809	0,0243	—0,9740
	10	12818,8	1,9805	—0,00827	0,0249	—1,0105
	∞	12879,3	1,9792	—0,00833	0,0251	—1,0359
60	5	40728,1	1,2992	—0,05128	0,2512	3,5609
	10	40922,2	1,2976	—0,05120	0,2515	3,3575
	∞	41116,9	1,2962	—0,05098	0,2512	3,1465
70	5	211752,5	0,7861	—0,1006	1,0314	171,40
	10	212727,8	0,7852	—0,1006	1,0343	170,07
	∞	213705,5	0,7844	—0,1006	1,0371	168,71
80	5	3452064,1	0,3726	—0,0195	0,7671	750,41
	10	3467521,1	0,3722	—0,0194	0,7659	744,09
	∞	3483012,6	0,3719	—0,0193	0,7647	737,69

$$\mathbf{w} = \left(- z\cos\varphi + \varepsilon\beta_1 \frac{\partial \Phi_0}{\partial z} + \varepsilon^2\beta_1^2 \frac{\partial w_{23}}{\partial z},\ - \varepsilon\beta_1 \frac{\partial \Phi_0}{\partial x} - \varepsilon^2\beta_1^2 \frac{\partial w_{23}}{\partial x}\right) + O(\varepsilon^3),$$

$$\tau = - z + B/C + \varepsilon\beta_1 T_0 + \varepsilon^2\beta_1^2 w_{22} + O(\varepsilon^3). \quad (3.3)$$

The functions $\psi_0(x, z)$, $T_0(x, z)$, $\Phi_0(x, z)$, $w_{21}(x, z)$, $w_{22}(x, z)$, $w_{23}(x, z)$, and also coefficient β_1 are expressed by solutions of linear boundary problems which arise in realizing the Lyapunov – Shmidt method [22].

$$\xi = - \sqrt{\frac{2\mu}{\Pr}}\,\mathbf{w}\sin\omega_2 t, \quad \eta = - \sqrt{\frac{2\mu}{\Pr}}\,(\mathbf{w}, \nabla\tau)\frac{\cos\omega_2 t}{\omega_2}$$

($\omega_2 = \omega\ell^2/\nu$ is dimensionless vibration frequency).

Solution (2.3) is used for calculating the intensity of transverse heat transfer (average Nusselt number <Nu>) and average kinetic energy ($\langle\|\mathbf{v}\|^2\rangle$) which in the vicinity of μ_* are calculated by the equations

$$\langle \mathrm{Nu}\rangle \equiv - \frac{\alpha}{2\pi}\int_0^{2\pi/\alpha} \frac{\partial T}{\partial z}\bigg|_{z=-1/2} dx = 1 + C_1\varepsilon_1 + O(\varepsilon^3); \quad (3.4)$$

$$\langle \|\mathbf{v}\|^2\rangle = \frac{\alpha\omega_2}{4\pi^2}\int_0^{2\pi/\omega_2}\int_0^{2\pi/\alpha}\int_{-0,5}^{0,5} \left|\mathbf{u} - \sqrt{\frac{2\mu}{\Pr}}\,\mathbf{w}\sin\omega_2 t\right|^2 dz\,dx\,dt = \frac{\mu_*\cos^2\varphi}{12}(1+\varepsilon_1) + C_2\varepsilon_1 + O(\varepsilon^3) \quad (3.5)$$

($\varepsilon_1 = \varepsilon^2/\mu_* = |\mu - \mu_*|/\mu_*$ is relative supercriticality). Expressions for C_1 and C_2 are given in [22] and $C_1 = C_2 = 0$ if $\mu < \mu_*$, $\delta_1 = 1$ and if $\mu > \mu_*$, $\delta_1 = -1$. The first term in (3.5) characterizes the contribution to kinetic energy of the rapid component ($\xi_0 = -\sqrt{2\mu/\Pr}\, w_0 \sin\omega_2 t$) of the main solution, and the second characterizes the contribution of addition terms ($\mathbf{v}_1$, $\xi_1 = -\sqrt{2\mu/\Pr}\,w_1 \sin\omega_2 t$) of the secondary flow velocity vector.

4. We give the main results of calculating with low gravitation ($g \to 0$) and pure weightlessness ($g = 0$). It is noted that the assertion of high-frequency vibration with $g \neq 0$ is

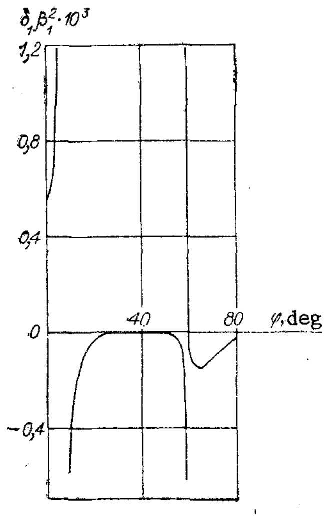

Fig. 1

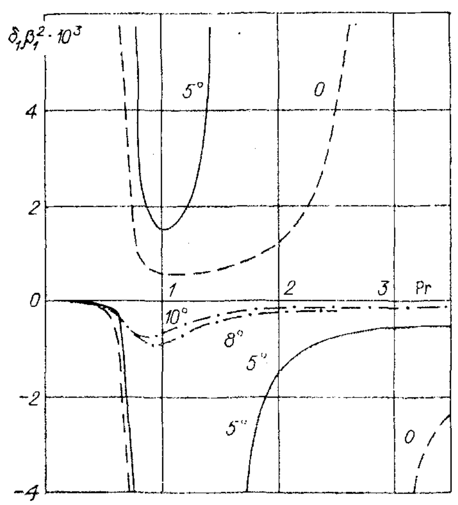

Fig. 2

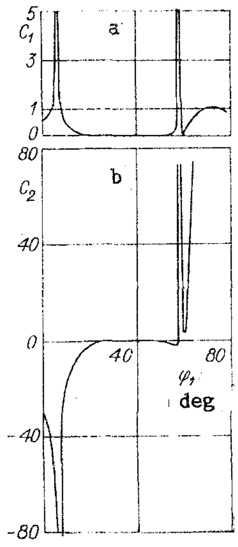

Fig. 3

TABLE 2

φ	μ_*	α_*	$\delta_1\beta_1^2$	C_1	C_2
6,26	2169,8	3,2192	0,19834	180,74	—10860
6,28	2170,1	3,2191	—0,15141	137,98	—8291,0
58,7	34 588	1,3743	—0,90167·10⁻³	4,1273	10,634
58,75	34 814	1,3712	0,11503·10⁻¹	5,2801	155,09
59,2	36 934	1,3439	0,19987·10⁻⁴	0,09407	0,58892
59,4	37 928	1,3318	—0,10243·10⁻⁴	0,04879	0,37947

described conveniently by parameter $r = \sqrt{\mu/R^2} = a\sqrt{\chi\nu}/(\sqrt{2}\,gl^3)$ which does not depend on temperature and it characterizes the ratio of vibration and gravitation forces. Thus, it appears [1, 16, 18] that in the case of vertical oscillations with r > 0.023 gravitation convection cannot arise with any temperature gradient.

In this work with fixed values of parameters r, φ, Pr we calculate $\mu_*(\alpha_*) = \min_\alpha \mu(\alpha)$, amplitudes β_1 of secondary solutions, and also C_1 and C_2 in (3.4) and (3.5) for <Nu> and the averaged square of the velocity norm $\langle\|v\|^2\rangle$ are calculated. It is noted that for flow function $\psi_0(x, z)$ the normalization $\psi_0(0, 0) = 4$ is taken.

From the results given in Table 1 it follows that a value of vibration parameter r_* ($r_* \approx 10$) exists such that with $r > r_*$ all of the movement characteristics calculated (β_1^2, $\langle Nu\rangle$, $\langle\|v\|^2\rangle$) correspond to the case of pure weightlessness. This makes it possible to select a vibration velocity with which terrestial modeling of weightlessness conditions is possible.

Presented in Fig. 1 is the dependence of the square of amplitude $\delta_1\beta_1^2(\varphi)$ for Pr = 1 and $\alpha = \alpha_*$. With vibration directions $0 < \varphi < 6.26°$ and $58.75° \leqslant \varphi \leqslant 59.2°$ secondary flow exists with supercritical branching μ ($\delta_1 = 1$), and with $6.28° \leqslant \varphi \leqslant 58.7°$ and $\varphi > 59.4°$ branching is subcritical ($\delta_1 = -1$); in the range $27° \leqslant \varphi \leqslant 50°$ secondary flow velocity is at a minimum ($\beta_1^2 \approx 10^{-6}$).

Shown in Fig. 2 is the dependence of the square of amplitude on Pr for some values of φ (0; 5; 8; 10°). With vibration directions close to longitudinal ($\varphi \approx 0$) there is a region of supercritical branching which decreases with an incresae in φ; with $\varphi \geqslant 8°$ branching is only subcritical. In addition, calculations showed that starting from Pr = 50 amplitude β_1 does not depend on Pr.

According to (3.4) and (3.5) in the small vicinity of μ_*, <Nu>, and $\langle\|v\|^2\rangle$ depend linearly on the relative supercriticality of $\varepsilon_1 = |\mu - \mu_*|/\mu_*$. It can be seen from Fig. 3a that heat transfer increases with an increase in supercriticality ($C_1 > 0$), and the rate of growth of C_1 depends markedly on vibration direction.

Presented in Fig. 3b are curves for $C_2(\varphi)$. With $\varphi \approx 58.3°$ coefficient C_2 changes sign, i.e., occurrence of secondary flow may reduce kinetic energy ($0 \leqslant \varphi \leqslant 58.2°$), but it may also increase ($\varphi \geqslant 58.4°$) which apparently makes it possible to control convection by means of a specially selected vibration direction.

Given in Table 2 are the results of calculations in the vicinity of values of φ, with which there is a change in the nature of branching: with $6.26° \leqslant \varphi \leqslant 6.28°$ and $58.7° \leqslant \varphi \leqslant 58.75°$ there is a strong increase in β_1^2, C_1, and C_2; in the section $59.2° \leqslant \varphi \leqslant 59.7°$ amplitude β_1^2 revert to zero, and C_1 and C_2 change weakly.

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